LIBRARY
TECHNICAL REPORT SECTION
NAVAL POSTGRADUATE SCHOOL
MONTEREY, CALIFORNIA 93940

NPS-69-78-012

NAVAL POSTGRADUATE SCHOOL

Monterey, California

A COMPUTER SUBROUTINE FOR STRESS ANALYSIS OF ROTATING, HEATED DISKS

by

John E. Brock

Robert E. Brown

May 1978

Approved for public release; distribution unlimited.

Prepared for:
Chief of Naval Research
Arlington, Virginia 22217

FEDDOCS
D 208.14/2:NPS-69-78-012

NAVAL POSTGRADUATE SCHOOL
Monterey, California

Rear Admiral Tyler Dedman
Superintendent

J. R. Borsting
Provost

A COMPUTER SUBROUTINE FOR STRESS ANALYSIS OF ROTATING, HEATED DISKS

This report gives listing and instructions for using a digital computer subroutine for finding stress distribution in a thin rotating disk with nonuniform heating; the problem is axisymmetric. An iterative method is used. Theoretical background is given.

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER NPS-69-78-012	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE *(and Subtitle)* A COMPUTER SUBROUTINE FOR STRESS ANALYSIS OF ROTATING, HEATED DISKS		5. TYPE OF REPORT & PERIOD COVERED 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) John E. Brock Robert E. Brown		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Professor John E. Brock (Code 69Bc) Department of Mechanical Engineering Naval Postgraduate School		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61152N, RR000-01-01 N0001478WR80023
11. CONTROLLING OFFICE NAME AND ADDRESS Naval Postgraduate School Monterey, California 93940		12. REPORT DATE May 1978 13. NUMBER OF PAGES
14. MONITORING AGENCY NAME & ADDRESS *(if different from Controlling Office)* Chief of Naval Research Arlington, Virginia 22217		15. SECURITY CLASS. *(of this report)* 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT *(of this Report)*

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT *(of the abstract entered in Block 20, if different from Report)*

18. SUPPLEMENTARY NOTES

19. KEY WORDS *(Continue on reverse side if necessary and identify by block number)*

Stress analysis
Rotating disks, Heated Disks
Axisymmetric Elasticity, Axisymmetric Disks, Elastic Disks

20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report gives listing and instructions for using a digital computer subroutine for finding stress distribution in a thin rotating disk with nonuniform heating; the problem is axisymmetric. An iterative method is used. Theoretical background is given.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE
S/N 0102-014-6601

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE *(When Data Entered)*

TABLE OF CONTENTS

A Computer Subroutine for Stress
Analysis of Rotating, Heated Disks
by
John E. Brock and Robert E. Brown

Introduction

Although, as is indicated by the title hereof, the principal purpose of this monograph is to present tested and proved digital computer software for the analysis of stress in a spinning axisymmetric disk having a radially variable thermal strain field, the opportunity is also taken of developing the theory and presenting some analytic solutions.

The method developed herein for computer analysis of disks having a general law of thickness variation was suggested by the algorithm contained in reference 2 and it appears to have advantages over such procedures as that of M. Donath, reference 3, which has been widely circulated in a book by S. Timoshenko, reference 5.

In what follows we immediately obtain a second order linear differential equation with dependent variable u, the radial deformation, and r, the radius. Analytic treatment is given for two particular laws of thickness variation. For the general case of thickness variation, the equation is recast as a second order linear differential equation in which the dependent variable is radial stress, σ_r. However, for numerical treatment an alternate form is more useful and direct and this forms the basis of the digital computer software which is given and illustrated in the appendices hereto.

Fundamental Analysis

We presume that the disk is thin enough and that the thickness varies slowly enough with respect to radius that we are justified in neglecting all stress components excepting only the radial stress σ_r and the circumferential stress σ_θ. Material properties E, Young's modulus, and ν, Poisson's ratio, are presumed to be indeed constant. The thermal strain field, αT, and the density γ may be specified functions of radius.

Two types of problem are considered:

1. Annular disk, $0 < a \leqq r \leqq b$, with $\sigma_r(a)$ and $\sigma_r(b)$ being specified.
2. Solid disk, $0 \leqq r \leqq b$, with $\sigma_r(b)$ being specified.

We also use the symbols $t = t(r)$ for disk thickness and ω for angular velocity. Other simplifying notation will be introduced later on.

Consideration of radial equilibrium leads without difficulty to the equation

$$\frac{1}{t}\frac{d(rt\sigma_r)}{dr} = \sigma_\theta - \gamma\omega^2 r^2 \qquad (1)$$

The thermoelastic constitutive equations are

$$E\varepsilon_\theta = \sigma_\theta - \nu\sigma_r + E\alpha T; \quad E\varepsilon_r = \sigma_r - \nu\sigma_\theta + E\alpha T \qquad (2a,b)$$

where the strain components are

$$\varepsilon_\theta = u/r; \quad \varepsilon_r = du/dr \qquad (3a,b)$$

Strain compatability leads to the equation

$$r\frac{d}{dr}(Eu/r) = -(1+\nu)(\sigma_\theta - \sigma_r) \qquad (4)$$

These equations may easily be combined into the differential equation

$$u'' + u'/r - u/r^2 + (t'/t)[u' + \nu u/r - (1+\nu)\alpha T]$$

$$= (1+\nu)(\alpha T)' - (1-\nu^2)\gamma\omega^2 r/E \qquad (5)$$

where primes denote differentiation with respect to r.

Two particular laws of thickness variation permit simple analytic treatment.

<u>Power Law of Thickness Variation</u>

If

$$t = t_o(r/r_o)^n \qquad (6)$$

so that

$$(t'/t) = n/r \qquad (7)$$

then equation 5 becomes

$$u'' + (1+\nu n)u'/r + (\nu n-1)u/r^2 = \beta' + n\beta/r - kr \qquad (8)$$

where

$$\beta = (1+\nu)\alpha T, \qquad k = (1-\nu^2)\gamma\omega^2/E \qquad (9,10)$$

Equation 8 may be rewritten as

$$\frac{d}{dr}[r^{1-m}\frac{d}{dr}(r^{(m+n)/2}\ u)] = r^{(2+n-m)/2}\ (\beta' + n\beta/r - kr) \qquad (11)$$

where

$$m = \pm\sqrt{(n^2-4\ n+4)} = \pm\sqrt{[(n-2)^2+4(1-\ )n]} \qquad (12)$$

and either the positive or the negative sign may be used Equation 11 may be proved simply by performing the indicated operations and comparing with equation 8.

The quantity on the right in equation 11 is well defined so that the solution of the differential equation may be obtained simply by integration, multiplication by r^{m-1}, and another integration. Two constants of integration are introduced. For the solid disk (case 2)

$u(0) = 0$ gives one condition and the second comes from satisfying the given value of $\sigma_r(b)$. For the annular disk (case 1) satisfying the given conditions $\sigma_r(a)$ and $\sigma_r(b)$ permits evaluating the constants.

An example with $n = -0.42$ is given in Appendix C. Note that the case $n = 0$ corresponds to a disk of uniform thickness.

Exponential Law of Thickness Variation

If

$$t = t_o \exp(-mr^2) \tag{13}$$

where m is a constant of appropriate dimensionality, then

$$(t'/t) = -2rm \tag{14}$$

and equation 5 becomes

$$u'' + (1/r - 2rm)u' - (1/r^2 + 2\nu m) = -2rm\beta + \beta' - kr \tag{15}$$

If additionally we assume that $\beta' = 0$ (which makes the thermal strain field constant — a triviality) the solution is simply

$$u = r(\beta + k/2m)/(1+\nu) \tag{16}$$

In this case we can easily find

$$\sigma_r = \sigma_\theta = \gamma\omega^2/2m \tag{17}$$

which is independent of r. Thus, if an allowable normal stress σ_A is specified and if blade or bucket loading at $r = b$ is w (pounds, say) per unit circumference, then a disk having thickness

$$t = (w/\sigma_A) \exp[(b^2 - r^2)(\gamma\omega^2/2\sigma_A)] \tag{18}$$

will be such that $\sigma_r \equiv \sigma_\theta \equiv \sigma_A$. If the failure criterion is the maximum shearing stress criterion (Tresca's condition), it is clear that this disk is optimal in the sense of having least volume and thus having least weight. If the failure criterion were that of von Mises, a slightly lighter disk would suffice.

General Law of Thickness Variation

Equations 3a and 2a give

$$Eu = rE\alpha T + r(\sigma_\theta - \nu\sigma_r) \tag{19}$$

and by differentiation

$$Eu' = E\alpha T + rE(\alpha T)' + (\sigma_\theta - \nu\sigma_r) + r(\sigma_\theta' - \nu\sigma_r') \tag{20}$$

From 3b and 2b we also have

$$Eu' = E\alpha T + \sigma_r - \nu\sigma_\theta \tag{21}$$

Subtracting 21 from 20 and rearranging gives

$$E(\alpha T)' + (1+\nu)(\sigma_\theta - \sigma_r)/r + \sigma_\theta' - \nu\sigma_r' = 0 \tag{22}$$

Equation 1 may be rewritten as

$$\sigma_\theta - \sigma_r = r\sigma_r' + rv\sigma_r + \gamma\omega^2 r^2 \tag{23}$$

where we have written

$$v = t'/t \tag{24}$$

for convenience. Differentiating 23 we get

$$\sigma_\theta' = 2\sigma_r' + r\sigma_r'' + v\sigma_r + rv\sigma_r' + rv'\sigma_r + 2\gamma\omega^2 r \tag{25}$$

and substituting 23 and 25 into 22 gives

$$r\sigma_r'' + (3+rv)\sigma_r' + [(2+\nu)v + rv']\sigma_r + (3+\nu)\gamma\omega^2 r + E(\alpha T)' = 0 \tag{26}$$

This is a single differential equation with dependent variable σ_r and can be dealt with by standard numerical methods. The conditions for evaluating the constants of integration have been mentioned earlier.

However, the preceding procedure is not particularly satisfactory. For one thing, the solid disk case for which r can become zero encounters numerical difficulties unless special treatment is employed to evade them. More importantly, however, if T is given by a graph or a numerical table, determination of $(\alpha T)'$ may involve numerical difficulties which, ultimately, are due to our having performed the differentiation to arrive at equation 20. Accordingly, an alternate procedure which adheres more closely to the fundamental mechanics of the problem is described in what follows.

We consider the annular disk first and we represent the unknown stress difference $\sigma_\theta - \sigma_r$ in the form

$$\sigma_\theta - \sigma_r = (A+B\eta)r \tag{27}$$

where A and B are unknown constants and η is an unknown function normalized so that

$$\eta(a) = 0, \quad \eta(b) = 1 \tag{28a,b}$$

Initially we make an assumption for η taking a linear variation in the absence of better information. By the use of some of the preceding equations we will be able to construct an improved form for η and will iterate until there is satisfactory convergence.

Let

$$z = Eu/r, \quad w = E\alpha T, \quad \beta = \gamma\omega^2 \tag{29,30,31}$$

noting that w and β have different meanings here than when they were used earlier. We can recast equation 1 as

$$d(t\sigma_r)/dr = (\sigma_\theta - \sigma_r)t/r - \gamma\omega^2 rt = t(A+B\eta-\gamma\omega^2 r) \tag{32}$$

Before performing the indicated integration we introduce two convenient notational devices, viz.

$$p\ldots = \int_0^r \ldots\, dr; \qquad *\ldots = [\ldots]_{r=b} \tag{33,34}$$

Thus, from equation 32 we have

$$t\sigma_r = (t\sigma_r)_a + Apt + Bp\eta t - p\beta rt \tag{35}$$

From equation 4 and equation 27 we have

$$dz/dr = -(1+\nu)(A+B\eta) \tag{36}$$

so that

$$z = (z)_a - (1+\nu)[A(r-a) + Bp\eta] \tag{37}$$

Equation 2a is

$$z = w + (\sigma_\theta-\sigma_r) + (1-\nu)\sigma_r = w + (A+B\eta)r + (1-\nu)\sigma_r \tag{38}$$

so that

$$(z)_a = (w)_a + Aa + (1-\nu)(\sigma_r)_a \tag{39}$$

$$(z)_b = (w)_b + (A+B)b + (1-\nu)(\sigma_r)_b \tag{40}$$

Evaluating 38 at $r = b$ and using 39 and 40 gives one equation involving the unknowns A and B. Evaluating 35 at $r = b$ gives a second. These equations can be arranged as

$$\begin{bmatrix} *(pt) & *(p\eta t) \\ (2+\nu)(b-a) & b+(1+\nu)*(p\eta) \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} *(p\beta rt) + (t\sigma_r)_b - (t\sigma_r)_a \\ (w)_a-(w)_b+(1-\nu)[(\sigma_r)_a-(\sigma_r)_b] \end{bmatrix} \tag{41}$$

so that one can easily solve for A and B. Then σ_r is obtained from 35, z is obtained from 37 and 39, and $(\sigma_\theta-\sigma_r)$ and σ_θ are obtained from 38. Then a new function η is calculated from

$$\eta = [(\sigma_\theta-\sigma_r)/r-A]/B \tag{42}$$

Using the new η the process is iterated, convergence being monitored by examination of the sequence of values of A and B that are calculated. When convergence is satisfactory, the desired functions σ_r and σ_θ are at hand.

The situation is simpler for the solid disk, case 2. $(\sigma_r)_a$ is not given but conditions of continuity require that $(\sigma_\theta-\sigma_r)/r$ vanish at $r = 0$. Thus A is zero and B can be obtained from

$$B = \frac{(w)_o - (w)_b + c[1-(t)_o/(t)_b](t\sigma_r)_b + c^*(p\beta rt)}{b + (1+\nu)^*(p\eta) + c^*(p\eta t)} \qquad (43)$$

where

$$c = (1-\nu)/(t)_o \qquad (44)$$

Otherwise the procedure is as for case 1.

Computer Implementation

The theory embodied in equations 27 through 44 and the associated procedure has been programmed for digital computer using the FORTRAN language. An initial programming based directly on the preceding equations and written in January 1978 by the junior author hereof has been supplanted by a newer programming which is somewhat more compact due to the employment therein of ancillary subroutines developed for use in another problem (the lateral buckling of elastic beams) on which we are working. This program, actually a subroutine called RODISK is listed in Appendix A hereof. This listing itself contains comments which adequately explain the construction of a MAIN program which supplies necessary input information and which invokes RODISK. Appendix B lists the ancillary subroutines. In each case comments indicate the purpose and employment of the subroutine. These may prove useful in constructing the MAIN program. For this reason, the ancillary subroutines DUPV and PRIV are given even through they are not called by RODISK.

In the computer implementation, the various *functions* of r which appear in the theory are represented by *vectors* the elements of which are function values at equally spaced values of r.

The ancillary subroutines manipulate these vectors. All of these are obvious except possibly INTV which performs an integration by use of Milne's formulas, cf. reference 4.

In the subroutine RODISK there is a slight departure from the theory as given herein. As a first step, all quantities and functions were "dedimensionalized" but otherwise the method is just as described above. Somewhat finer details of what was actually done may be found in reference 1.

Appendix C contains some examples and remarks concerning them.

References

1. Brock, J. E., *Stress analysis of axisymmetric rotating disks,* submitted for publication.
2. Brock, J. E., *A method for analyzing axisymmetric plates with complicating conditions,* J. Appl. Mech., Vol. 29, March 1962, pp. 1 - 6.
3. Donath, M., *Die Berechnung rotierender Scheiben und Ringe,* Berlin, 1912.
4. Milne, W. E., *Numerical Calulus*, Princeton Univ. Press, 1949, p. 119.
5. Timoshenko, S. *Strength of Materials, Part II,* 3rd ed., D. van Nostrand Co., Inc., 1956, pp. 223 - 228.

Appendix A

Listing of subroutine RODISK

(The listing on this page, page 10, is of the comments which provide instructions for the use of RODISK. Commands appear on the following two pages.)

```
SUBROUTINE RODISK.  JOHN E. BROCK., 1 MAY 1978
THIS IS A SUBROUTINE FOR DETERMINING RADIAL AND CIRCUM-
FERENTIAL STRESSES IN AN AXISYMMETRIC THIN ELASTIC DISK,
ROTATING AT ANGULAR VELOCITY OMEGA (RADIANS PER SECOND)
ABOUT ITS AXIS OF SYMMETRY AND HAVING AN AXISYMMETRIC
DISTRIBUTION OF THERMAL STRAIN.  TWO TYPES OF PROBLEM MAY
BE TREATED:
         TYPE 1.  ANNULAR DISK OF INSIDE RADIUS ARAD AND
         OUTSIDE RADIUS BRAD.  THE RADIAL STRESS IS SRA AT THE
         INNER RADIUS AND SRB AT THE OUTER RADIUS.  THE INSIDE
         RADIUS MUST BE GREATER THAN ZERO.
         TYPE 2.  SOLID DISK OF OUTSIDE RADIUS BRAD AND WITH
         RADIAL STRESS SRB AT THE OUTSIDE RADIUS.
THE USER MUST PROVIDE A MAIN PROGRAM WHICH CALLS SUBROUTINE
RODISK AFTER IT HAS SUPPLIED THE FOLLOWING INFORMATION.
  (1)    N, INTEGER.  (N-1) IS THE NUMBER OF EQUAL SUBDIVISIONS
         INTO WHICH THE ANNULAR RADIUS (BRAD MINUS ARAD) IS
         DIVIDED FOR COMPUTATIONAL PURPOSES.  THE PRESENT
         DIMENSIONING CAN ACCOMMODATE N NOT LARGER THAN 101.
  (2)    BRAD
  (3)    ARAD  (NOT NECESSARY FOR PROBLEMS OF TYPE 2.)
  (4)    SRB
  (5)    SRA  (NOT NECESSARY FOR PROBLEMS OF TYPE 2.)
  (6)    TEEBEE, DISK THICKNESS AT OUTSIDE RADIUS
  (7)    POIS, POISSON'S RATIO.
  (8)    KP(1) = 1,2.  INTEGER TO DENOTE PROBLEM OF TYPE 1,2.
  (9)    KP(2), INTEGER TO PROVIDE FOR SKIPPING WHILE PRINTING
         OUTPUT.  FOR EXAMPLE, IF N=101 AND KP(2)=5, ONLY
         EVERY FIFTH SET OF VALUES WILL BE PRINTED: 1ST, 6TH,
         ..., 96TH, 101ST.
 (10)    KP(3), INTEGER SPECIFYING THE NUMBER OF ITERATIONS
         TO BE PERFORMED.  USUALLY KP(3)=10 IS SUFFICIENT FOR
         ENGINEERING ACCURACY.
 (11)    KP(4).  IF KP(4)=0 ONLY FINAL ANSWERS WILL BE PRINTED
         IF KP(4)=1 A SEQUENCE OF ITERANT VALUES WILL BE
         PRINTED, INDICATING DEGREE OF CONVERGENCE.
 (12)    VECTORS X(I,J), I=1,2,3; J=1,2,...,N.
         VECTOR X(1,J) CONTAINS VALUES OF THE RATIO (LOCAL
THICKNESS OF DISK)/(TEEBEE) COMPUTED AT EQUALLY SPACED
RADII STARTING AT THE INSIDE AND ENDING AT THE OUTSIDE.
         VECTOR X(2,J) CONTAINS VALUES OF (GAMMA)(OMEGA-SQUARE
TIMES (BRAD-SQARED) DIVIDED BY (SRB).  FOR MOST PROBLEMS
GAMMA DOES NOT VARY WITH RADIUS AND THIS QUANTITY IS A
CONSTANT.
         VECTOR X(3,J) CONTAINS VALUES OF (E)(ALPHA)(TEE)/(SRB
WHERE (E) IS YOUNG'S MODULUS, (ALPHA) IS THE COEFFICIENT O
LINEAR THERMAL EXPANSION, AND (TEE) IS TEMPERATURE CHANGE.
         THE MAIN PROGRAM MUST CONTAIN THE STATEMENTS:
IMPLICIT REAL*8 (A-H,O-Z)
INTEGER KP(4)
COMMON X,N
COMMON /ONE/ARAD,BRAD,SRA,SRB,TEEBEE,POIS,KP
         FOLLOWING SUBROUTINE RODISK THERE ARE SEVERAL
ANCILLARY SUBROUTINES WHICH PERFORM VARIOUS OPERATIONS
ON THE VECTORS X(I,J).  THE FUNCTION OF EACH IS OBVIOUS
FROM THE LISTING.  THEY MAY BE USED IN THE USER'S MAIN
PROGRAM.  TWO OF THESE ANCILLARY SUBROUTINES WHICH ARE
AVAILABLE IN THIS PACKAGE BUT WHICH ARE NOT CALLED BY
SUBROUTINE RODISK ARE DUPV WHICH DUPLICATES A VECTOR AND
PRIV WHICH PRINTS A VECTOR.
```

```
      SUBROUTINE RODISK
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 X(20,101)
      INTEGER KP(4)
      COMMON X,N
      COMMON /ONE/ARAD,BRAD,SRA,SRB,TEEBEE,POIS,KP
      ONE=1.D+0
      POIS=3.D-1
      ZERO=C.D+0
      IF(KP(1).EQ.2) ARAD=ZERO
      RHO=ARAD/BRAD
      ENM=N-1
      WRITE(6,2) KP(1)
    2 FORMAT(//,10X,'RODISK PROBLEM OF TYPE ',I1,'.',//)
      DO 5 I=1,N
      EIM=I-1
      Y=EIM/ENM
      X(4,I)=RHO+(ONE-RHO)*Y
      X(5,I)=Y
    5 X(6,I)=Y
      ITER=1
      ETA=X(1,1)/X(1,N)
      IF(KP(1).EQ.2) GO TO 100
C     THE PROBLEM IS OF TYPE ONE: ANNULAR DISK
      SRAT=SRA/SRB
      C1=(2.D+0+POIS)*(ONE-RHO)
      CALL INTV(1,7)
      C2=X(7,N)
      C5=X(3,1)-X(3,N)-(ONE-POIS)*(ONE-SRAT)
      CALL MULV(1,2,8)
      CALL MULV(8,4,9)
      CALL INTV(9,10)
      C6=X(10,N)+(ONE-ETA*SRAT)/(ONE-RHO)
   20 CALL INTV(6,11)
      C3=ONE+(ONE-RHO)*(ONE+POIS)*X(11,N)
      CALL MULV(1,6,12)
      CALL INTV(12,13)
      C4=X(13,N)
      D=C1*C4-C2*C3
      A=(C5*C4-C6*C3)/D
      B=(C1*C6-C2*C5)/D
      IF(KP(4).EQ.1) WRITE(6,7) ITER,A,B
    7 FORMAT(5X,I10,1P2E20.5)
      CALL MULS(7,14,A)
      CALL MULS(13,15,B)
      CALL ADDV(14,15,15)
      CALL SUBV(15,10,16)
      S=ONE-RHO
      CALL MULS(16,16,S)
      S=ETA*SRAT
      CALL ADDS(16,16,S)
      CALL DIVV(16,1,20)
      Z0=A*RHO+X(3,1)+(ONE-POIS)*SRAT
      CALL MULS(11,11,B)
      CALL MULS(5,16,A)
      CALL ADDV(11,16,16)
      S=-(ONE-RHO)*(ONE+POIS)
      CALL MULS(16,16,S)
      CALL ADDS(16,19,Z0)
      S=POIS-ONE
      CALL MULS(20,18,S)
      CALL ADDV(18,19,18)
      CALL SUBV(18,3,19)
      ITER=ITER+1
      IF(ITER.GT.KP(3)) GO TO 200
      CALL DIVV(19,4,17)
      CALL SUBS(17,17,A)
      CALL DIVS(17,6,B)
      GO TO 20
```

```
100 C1=POIS-ONE
    C2=ETA
    C5=X(3,1)-X(3,N)+POIS-ONE
    CALL MULV(1,2,7)
    CALL MULV(7,4,8)
    CALL INTV(8,9)
    C6=ONE+X(9,N)
105 CALL INTV(6,10)
    C3=ONE+(ONE+POIS)*X(10,N)
    CALL MULV(1,6,11)
    CALL INTV(11,12)
    C4=X(12,N)
    D=C1*C4-C2*C3
    SRAT=(C5*C4-C6*C3)/D
    B=(C1*C6-C2*C5)/D
    IF(KP(4).EQ.1) WRITE(6,7) ITER,SRAT,B
    CALL MULS(12,13,B)
    CALL SUBV(13,9,14)
    S=ETA*SRAT
    CALL ADDS(14,20,S)
    CALL DIVV(20,1,20)
    S=-B*(ONE+POIS)
    CALL MULS(10,15,S)
    S=POIS-ONE
    CALL MULS(20,18,S)
    S=X(3,1)+(ONE-POIS)*SRAT
    CALL ADDS(18,19,S)
    CALL SUBV(19,3,19)
    S=-B*(ONE+POIS)
    CALL MULS(10,17,S)
    CALL ADDV(19,17,19)
    ITER=ITER+1
    IF(ITER.GT.KP(3)) GO TO 200
    CALL MULS(4,16,B)
    X(16,1)=ONE
    CALL DIVV(19,16,6)
    GO TO 105
200 CALL MULS(4,7,BRAD)
    CALL MULS(20,8,SRB)
    CALL MULS(19,9,SRB)
    CALL ADDV(8,9,9)
    S=SRB/BRAD**2
    CALL MULS(2,10,S)
    CALL MULS(3,11,SRB)
    CALL MULS(1,15,TEEBEE)
    WRITE(6,204)
204 FORMAT(////)
    WRITE(6,205)
205 FORMAT(24X,'RADIUS',11X,'THICKNESS',6X,'GAMMA.OMEGA.SQ'
   1'EE.ALPHA.TEE',8X,'SIGMA.RADIAL',7X,'SIGMA.CIRCUMF')
    KSKIP=KP(2)
    DO 210 I=1,N,KSKIP
    J=I/KSKIP
    WRITE(6,211)J,X(7,I),X(15,I),X(10,I),X(11,I),X(8,I),X(9
210 CONTINUE
211 FORMAT(I10,1P6E20.5)
    RETURN
    END
```

Appendix B

Listing of ancillary subroutines

```
      SUBROUTINE ADDV(N1,N2,N3)
C        ANCILLARY SUBROUTINE
C        ADD TWO VECTORS TERM BY TERM
C        X(N3,I)=X(N1,I)+X(N2,I)
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N3,I)=X(N1,I)+X(N2,I)
      RETURN
      END

      SUBROUTINE SUBV(N1,N2,N3)
C        ANCILLARY SUBROUTINE
C        SUBTRACT TWO VECTORS TERM BY TERM
C        X(N3,I)=X(N1,I)-X(N2,I)
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N3,I)=X(N1,I)-X(N2,I)
      RETURN
      END

      SUBROUTINE MULV(N1,N2,N3)
C        ANCILLARY SUBROUTINE
C        MULTIPLY TWO VECTORS TERM BY TERM
C        X(N3,I)=X(N1,I)*X(N2,I)
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N3,I)=X(N1,I)*X(N2,I)
      RETURN
      END

      SUBROUTINE DIVV(N1,N2,N3)
C        ANCILLARY SUBROUTINE
C        DIVIDE TWO VECTORS TERM BY TERM
C        X(N3,I)=X(N1,I)/X(N2,I)
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N3,I)=X(N1,I)/X(N2,I)
      RETURN
      END

      SUBROUTINE ADDS(N1,N2,S)
C        ANCILLARY SUBROUTINE
C        ADD A SCALAR TO EACH TERM OF A VECTOR
C        X(N2,I)=X(N1,I)+S
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N2,I)=X(N1.I)+S
      RETURN
      END
```

```
      SUBROUTINE SUBS(N1,N2,S)
C         ANCILLARY SUBROUTINE
C         SUBTRACT A SCALAR FROM EACH TERM OF A VECTOR
C         X(N2,I)=X(N1,I)-S
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N2,I)=X(N1,I)-S
      RETURN
      END

      SUBROUTINE MULS(N1,N2,S)
C         ANCILLARY SUBROUTINE
C         MULTIPLY EACH TERM OF A VECTOR BY A SCALAR
C         X(N2,I)=X(N1,I)*S
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N2,I)=X(N1,I)*S
      RETURN
      END

      SUBROUTINE DIVS(N1,N2,S)
C         ANCILLARY SUBROUTINE
C         DIVIDE EACH TERM OF A VECTOR BY A SCALAR
C         X(N2,I)=X(N1,I)/S
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N2,I)=X(N1,I)/S
      RETURN
      END

      SUBROUTINE DUPV(N1,N2)
C         ANCILLARY SUBROUTINE
C         DUPLICATE A VECTOR
C         X(N2,I)=X(N1,I)
      REAL*8 X(20,101),S
      COMMON X,N
      DO 1 I=1,N
    1 X(N2,I)=X(N1,I)
      RETURN
      END

      SUBROUTINE INTV(N1,N2)
C         ANCILLARY SUBROUTINE
C         INTEGRATE A VECTOR USING MILNE'S METHOD
      REAL*8 X(20,101),EN,R,ADD,NINO,NTNO,FIVO,THTO
      COMMON X,N
      EN=N-1
      EN=1.D+0/EN
      R=EN/2.4D+1
      NINO=R*9.D+0
      NTNO=R*1.9D+1
      FIVO=R*5.D+0
      THTO=R*1.3D+1
      X(N2,1)=0.D+0
      X(N2,2)=NINO*X(N1,1)+NTNO*X(N1,2)-FIVO*X(N1,3)+R*X(N1,4
      NM3=N-3
      DO 1 K=1,NM3
      KP1=K+1
      KP2=K+2
      KP3=K+3
       ADD=THTO*(X(N1,KP1)+X(N1,KP2))-R*(X(N1,K)+X(N1,KP3))
    1 X(N2,KP2)=X(N2,KP1)+ADD
      X(N2,N)=X(N2,N-1)+NINO*X(N1,N)+NTNO*X(N1,N-1)-FIVO*X(N1
     1+R*X(N1,N-3)
      RETURN
      END
```

```
      SUBROUTINE PRIV(N1,I,J)
C        ANCILLARY SUBROUTINE
C        PRINT A VECTOR
      REAL*8 X(20,101),S
      COMMON X,N
C   IF SECOND ARGUMENT EQUALS 0, GO DIRECTLY TO RETURN
C   IF SECOND ARGUMENT EQUALS 1,PRINT THE VECTOR.
C   IF SECOND ARGUMENT EQUALS 2, PRINT THE IDENTITY AND THE VECTOR.
C   IF SECOND ARGUMENT EQUALS 3, PRINT THE VECTOR NUMBER AND THE VECTOR.
C   IF SECOND ARGUMENT EQUALS 4, PRINT NUMBER, IDENTITY, AND VECTOR.
C   IF SECOND ARGUMENT EQUALS 5, PRINT IDENTITY ONLY.
      IF(I.EQ.0) GO TO 10
      IF(I.EQ.1) GO TO 1
      IF(I.EQ.2) GO TO 2
      IF(I.EQ.3) GO TO 3
      IF(I.EQ.4) GO TO 4
      IF(I.EQ.5) GO TO 5
    1 DO 8 K=1,N
    8 WRITE(6,9) K,X(N1,K)
    9 FORMAT(30X, I5,1PE20.5)
   10 RETURN
    2 WRITE(6,21) J
   21 FORMAT(//,30X,'VECTOR WITH IDENTITY ',I5,' FOLLOWS:')
      GO TO 1
    3 WRITE(6,31) N1
   31 FORMAT(//,30X,'VECTOR NUMBER ',I5,' FOLLOWS:')
      GO TO 1
    4 WRITE(6,41) N1,J
   41 FORMAT(//,30X,'VECTOR X',I2,' WITH IDENTITY ',I5,' FOLLOWS:')
      GO TO 1
    5 WRITE(6,51) J
   51 FORMAT(//,30X,'VECTOR WITH IDENTITY ',I5,' HAS BEEN GENERATED.')
      GO TO 10
      END
```

Appendix C

Examples

The listing on the next page is of a MAIN program which calls RODISK twice to solve two different problems. On an IBM 360/67 compile time (for the MAIN, RODISK, and the ancillary subroutines) was 12 s, link time was 2 s, and execution time for both broblems was 1.5 s. In both problems we used N = 41 and iterated 10 times.

The first problem is of type 2 (solid disk) with b = 10, $t = 1/(1.6+.008r^2)$, $\nu = 0.3$, $\gamma\omega^2 = 120$, $\sigma_r(b) = 14000$, and $E\alpha T = 25105 + 1300 \log_e(t) - r^2(233+16t)$ Units are inches and pounds. The problem was made up from the exact solution

$$\sigma_r = 9000 + 50r^2; \qquad \sigma_\theta = \sigma_r + r^2(120+16t)$$

The results, shown on page 18, show evaluations for the stress components which are correct within 0.03 psi even though convergence was complete to only about 4 digits as indicated by the sequence of values above the final tabulation; these are values of $\sigma_r(0)/\sigma_r(b)$ and of B.

The second problem is of type 1 with a = 2.57, b = 5.15, $\sigma_r(a) = 18205$, $\sigma_r(b) = 22000$, $t = 0.1493\ r^{-.42}$, $T = 60 - 1.6r^2$, $\nu = 0.3$, and $E\alpha = 194.3$. The units are inches and pounds. Since the thickness variation is a power law, the theoretical solution given in the body hereof may be used to obtain the theoretically exact solution

$$\sigma_r = -113.95r^2 + 15832r^p - 11708r^q$$

$$\sigma_\theta = 122.80r^2 + 13801r^p + 14310r^q$$

```
C          J. E. BROCK    1 MAY 1978
C          THIS IS A MAIN PROGRAM TO TEST MY SUBROUTINE RODISK
      IMPLICIT REAL*8(A-H,O-Z)
      REAL*8 X(20,101)
      INTEGER KP(4)
      COMMON X,N
      COMMON /ONE/ARAD,BRAD,SRA,SRB,TEEBEE,POIS,KP
C          THE FIRST PROBLEM IS FOR A SOLID DISK
      KP(1)=2
      KP(2)=4
      KP(3)=10
      KP(4)=1
      BRAD=1.D+1
      SRB=1.4D+4
      N=41
      ENM=N-1
      DO 20 I=1,N
      EIM=I-1
      R=EIM*BRAD/ENM
      THICK=1.D+0/(1.6D+0+8.D-3*R**2)
      X(20,I)=THICK
      X(2,I)=120.D+0*BRAD**2/SRB
      W=2.5105D+4+1.3D+3*DLOG(THICK)-(R**2)*(2.33D+2+1.6D+1*THICK
      X(3,I)=W/SRB
   20 CONTINUE
      S=X(20,N)
      TEEBEE=S
      CALL DIVS(20,1,S)
      CALL RODISK
C          THE SECOND PROBLEM IS FOR AN ANNULAR DISK
      TEEBEE=1.493D-1*5.15D+0**(-4.2D-1)
      ENM=N-1
      KP(1)=1
      ARAD=2.57D+0
      BRAD=5.15D+0
      SRA=1.8205D+4
      SRB=2.2D+4
      BETA=5.02473471D-1
      DO 30 I=1,N
      EIM=I-1
      Y=EIM/ENM
      X(5,I)=Y
      X(6,I)=Y
      R=ARAD+(BRAD-ARAD)*Y
      X(4,I)=R/BRAD
      THICK=1.493D-1*R**(-4.2D-1)
      X(1,I)=THICK/TEEBEE
      X(2,I)=BETA
      X(3,I)=(6.D+1-1.6D+0*R**2)*1.943D+2/SRB
   30 CONTINUE
      CALL RODISK
      STOP
      END
```

RODISK PROBLEM OF TYPE 2.

1	6.45294D-01	9.09855D-01
2	6.41506D-01	9.01924D-01
3	6.43559D-01	9.06243D-01
4	6.42515D-01	9.04038D-01
5	6.43018D-01	9.05102D-01
6	6.42784D-01	9.04608D-01
7	6.42889D-01	9.04830D-01
8	6.42843D-01	9.04733D-01
9	6.42863D-01	9.04774D-01
10	6.42855D-01	9.04757D-01

	RADIUS	THICKNESS	GAMMA.OMEGA.SQ	EE.ALPHA.TEE	SIGMA.RADIAL	SIGMA.CIRCUMF
0	0.0	6.25000D-01	1.20000D 02	2.44940D 04	8.99997D 03	8.99997D 03
1	1.00000D 00	6.21891D-01	1.20000D 02	2.42446D 04	9.04997D 03	9.17992D 03
2	2.00000D 00	6.12745D-01	1.20000D 02	2.34970D 04	9.19997D 03	9.71916D 03
3	3.00000D 00	5.98086D-01	1.20000D 02	2.22536D 04	9.44997D 03	1.05161D 04
4	4.00000D 00	5.78704D-01	1.20000D 02	2.05178D 04	9.79998D 03	1.16681D 04
5	5.00000D 00	5.55556D-01	1.20000D 02	1.82937D 04	1.02500D 04	1.34722D 04
6	6.00000D 00	5.29661D-01	1.20000D 02	1.55857D 04	1.08000D 04	1.54250D 04
7	7.00000D 00	5.02008D-01	1.20000D 02	1.23985D 04	1.14500D 04	1.77255D 04
8	8.00000D 00	4.73485D-01	1.20000D 02	8.73623D 03	1.22000D 04	2.03646D 04
9	9.00000D 00	4.44840D-01	1.20000D 02	4.60243D 03	1.30500D 04	2.33464D 04
10	1.00000D 01	4.16667D-01	1.20000D 02	2.23975D-01	1.40000D 04	2.66666D 04

RODISK PROBLEM OF TYPE 1.

1	2.11476D-01	-6.25861D-02
2	3.93726D-01	-1.53012D-01
3	2.69932D-01	-9.19338D-02
4	3.04670D-01	-1.09032D-01
5	2.90174D-01	-1.01896D-01
6	2.95421D-01	-1.04480D-01
7	2.93395D-01	-1.03482D-01
8	2.94162D-01	-1.03860D-01
9	2.93870D-01	-1.03716D-01
10	2.93981D-01	-1.03771D-01

	RADIUS	THICKNESS	GAMMA.OMEGA.SQ	EE.ALPHA.TEE	SIGMA.RADIAL	SIGMA.CIRCUMF
0	2.57000D 00	1.00436D-01	4.16794D 02	9.60467D 03	1.82050D 04	2.14325D 04
1	2.82800D 00	9.64803D-02	4.16794D 02	9.17171D 03	1.89467D 04	2.17170D 04
2	3.08600D 00	9.30065D-02	4.16794D 02	8.69737D 03	1.95646D 04	2.20783D 04
3	3.34400D 00	8.99224D-02	4.16794D 02	8.18164D 03	2.00843D 04	2.24941D 04
4	3.60200D 00	8.71588D-02	4.16794D 02	7.62452D 03	2.05237D 04	2.29503D 04
5	3.86000D 00	8.46629D-02	4.16794D 02	7.02601D 03	2.08956D 04	2.34377D 04
6	4.11800D 00	8.23932D-02	4.16794D 02	6.38612D 03	2.12092D 04	2.39502D 04
7	4.37600D 00	8.03170D-02	4.16794D 02	5.70484D 03	2.14716D 04	2.44836D 04
8	4.63400D 00	7.84076D-02	4.16794D 02	4.98218D 03	2.16881D 04	2.50349D 04
9	4.89200D 00	7.66435D-02	4.16794D 02	4.21812D 03	2.18631D 04	2.56024D 04
10	5.15000D 00	7.50068D-02	4.16794D 02	3.41269D 03	2.20000D 04	2.61646D 04

where p = .29171 and q = -1.87171. The computer results agree with five digit accuracy even though the sequence of iterants (A,B) shows only four digit convergence.

The same two problems were also worked with N = 101 and 14 iterations. The execution time went from 1.5 s to 3.0 s. The maximum change in any stress value was 0.3 psi. From these and other problems it may be concluded that there are no difficulties of accuracy, computer storage, or execution time.

Acknowledgment

Appreciation is expressed for partial support by the Naval Postgraduate School Research Foundation.

INITIAL DISTRIBUTION LIST

1.	Defense Documentation Center Cameron Station Alexandria, VA 22314	2
2.	Library Naval Postgraduate School Monterey, CA 93940	2
3.	Dean of Research, Code 012 Naval Postgraduate School Monterey, CA 93940	1
4.	Chairman Department of Mechanical Engineering Naval Postgraduate School Monterey, CA 93940	1
5.	Professor J. E. Brock, Code 69Bc Naval Postgraduate School Monterey, CA 93940	15
6.	LCDR Robert E. Brown, USN Naval Amphibious School Coronado San Diego, CA 92155	2
7.	Mr. Charles Miller NAVSEA Code 0331 Naval Ship Systems Command Washington, DC 20362	1
8.	Mr. Richard Carleton NAVSEC Code 6146 Naval Ship Engineering Center Washington, DC 20362	1

U182836

DUDLEY KNOX LIBRARY - RESEARCH REPORTS
5 6853 01068819 5